Math 45 SSM 2/e 5.1. Adding and Subtracting Polynomials

Objectives: 1) What is a monomial?
Find the degree of a monomial
2) What is a polynomial?
Find the degree of a polynomial

Objectives: 3) Simplify polynomials by adding like terms
4) Simplify polynomials by distributing negative and adding like terms.
5) Evaluate polynomials at given value

Monomial: a product of a constant and/or one or more variables, each raised to a whole number exponent.
[Recall: $\mathbb{W}$ = whole numbers = $\{0, 1, 2, 3 \ldots\}$].

Polynomial: a monomial or the sum of monomials

Special words: binomial: 2 terms
trinomial: 3 terms.

Standard form of a polynomial is when highest degree term is first, in descending order.

The degree of monomial or polynomial is related to the exponents within it.

To find the degree of a monomial

Step 1: Identify all variables and their exponents, including invisible exponent 1.

Step 2: Add the exponents. The result is the degree.

To find the degree of a polynomial

Step 1: Identify each term of the polynomial.

Step 2: For each monomial term, find the degree (above).

Step 3: The degree of the polynomial is the largest result from step 2.

Examples: a) Identify if monomial, polynomial, or neither
b) If monomial or polynomial, find degree.

① $2x^4$

monomial
degree 4

② $-\frac{7}{4}x^2y$ $\quad x^2 \cdot y^1$

monomial
degree 3

$2+1=3$

③ $3x^{-2}$

neither

-2 is not a whole number

④ $5x^4y^2$

monomial
degree 6

$4+2=6$.

⑤ $x^2 - 8x + 3$

polynomial

deg 2

x^2 deg 2
$-8x$ deg 1
3 deg 0

max = 2

⑥ $7x^3 - 2x^2 + 6x + 4$

polynomial

deg 3

$7x^3$ deg 3
$-2x^2$ deg 2
$6x$ deg 1
4 deg 0

max = 3

⑦ $x^3y^4 - 3x^3y^2 + 2x^3y$

polynomial

deg 7

$x^3y^4 \to 3+4 \Rightarrow$ deg 7
$-3x^3y^2 \to 3+2 \Rightarrow$ deg 5
$2x^3y \to 3+1 \Rightarrow$ deg 4
} max = 7

Simplify completely

⑧ $(-5x^3 + 6x^2 + 2x - 7) + (3x^3 - 4x + 1)$

Note: The ( ) do not mean any mathematical operation - when there is a plus sign in between.

Note: But inside the parentheses, polynomials are usually written in standard form, which helps you organize the like terms.

$= -5x^3 + 6x^2 + 2x - 7$
$\quad + 3x^3 \qquad - 4x + 1$ vertical method.

$= \boxed{-2x^3 + 6x^2 - 2x - 6}$ combine like terms.

⑨ $(6a^2b - 4ab + 11ab^2) + (a^2b + 9ab - 3ab^2)$

$= 6a^2b - 4ab + 11ab^2$
$\quad + a^2b + 9ab - 3ab^2$

← Notice: The exponents on a decrease. So it's standard form.

$= \boxed{7a^2b + 5ab + 8ab^2}$

⑩ $(6z^3 + 2z^2 - 5) - (-3z^3 + 9z^2 - z + 1)$

Note: The second set of parentheses can't be removed until you distribute the negative.

$= 6z^3 + 2z^2 - 5 + 3z^3 - 9z^2 + z - 1$

$= \boxed{9z^3 - 7z^2 + z - 6}$

Simplify completely.

(11) $(2p^2 + pq + 5pq^2) - (3p^2q - 6pq + 9pq^2)$

$= 2p^2 + pq + 5pq^2$
$\quad + 6pq - 9pq^2 - 3p^2q$

} align like terms vertically
← dist negative

$= \boxed{2p^2 + 7pq - 4pq^2 - 3p^2q}$

(12) $(3x^2 - 2xy + y^2) - (7x^2 - y^2) + (3xy - 5y^2)$

↑ dist neg

$= 3x^2 - 2xy + y^2$
$-7x^2 \qquad + y^2$
$\quad + 3xy - 5y^2$

} align like terms vertically

$= \boxed{-4x^2 + xy - 3y^2}$

(13) $\left(\frac{5}{3}q^2 - \frac{5}{2}q + 4\right) - \left(\frac{1}{9}q^2 + \frac{3}{8}q + 2\right)$

$= \frac{5}{3}q^2 - \frac{5}{2}q + 4$
$-\frac{1}{9}q^2 - \frac{3}{8}q - 2$

$= \boxed{\frac{14}{9}q^2 - \frac{23}{8}q + 2}$

Scratch work:

$3\cdot\frac{5}{3} - \frac{1}{9}$ | $-\frac{5\cdot 1}{2\cdot 4} - \frac{3}{8}$

$= \frac{15-1}{9}$ | $= \frac{-20-3}{8}$

$= \frac{14}{9}$ | $= -\frac{23}{8}$

(14) $(4a^2 - 1) + (a^2 + 5a + 2) - (-a^2 + 4)$

$= 4a^2 \qquad - 1$
$\quad a^2 + 5a + 2$
$+ \quad a^2 \qquad - 4$

} align vertically
write each polynomial in standard form.

$\boxed{6a^2 + 5a - 3}$

Evaluate.

(15) $2x^3 - 5x^2 + x - 3$ for $x = 2$.

$2(2)^3 - 5(2)^2 + (2) - 3$ replace all x by (2).

$= 2 \cdot 8 - 5 \cdot 4 + 2 - 3$ exponents

$= 16 - 20 + 2 - 3$ multiply

$= \boxed{-5}$ ... add & subtract L→R.

(16) $2x^3 - 5x^2 + x - 3$ for $x = -2$

$2(-2)^3 - 5(-2)^2 + (-2) - 3$ replace all x by (-2)
Parentheses essential

$= 2(-8) - 5(4) - 2 - 3$ exponents

$= -16 - 20 - 2 - 3$ multiply

$= \boxed{-41}$ subtract L→R.

(17) $2a^2b + 3a^2b^2 - 4ab$ for $a = -3$ and $b = -1$.

$2(-3)^2(-1) + 3(-3)^2(-1)^2 - 4(-3)(-1)$ replace a by (-3)
replace b by (-1)
parentheses essential

$= 2(9)(-1) + 3(9)(1) - 4(-3)(-1)$ exponents

$= -18 + 27 - 12$ multiply

$= \boxed{-3}$ add & subtract L→R

(18) $2 + \frac{1}{2}n^2$ for $n = 4$

$2 + \frac{1}{2}(4)^2$ replace n by (4)

$= 2 + \frac{1}{2}(16)$ exponent

$= 2 + 8$ multiply

$= \boxed{10}$ add.

(19) $2+\frac{1}{2}n^2$ for $n=0.5$

$2+\frac{1}{2}(.5)^2$ replace n by (0.5) or $(\frac{1}{2})$

$=2+\frac{1}{2}(.25)$ exponent

$=2+(.125)$ multiply

$=\boxed{2.125}=\boxed{2\frac{1}{8}}=\boxed{\frac{17}{8}}$

(20) $2+\frac{1}{2}n^2$ for $n=-\frac{1}{4}$

$2+\frac{1}{2}(-\frac{1}{4})^2$ replace n by $(-\frac{1}{4})$

* parentheses essential *

$=2+\frac{1}{2}(\frac{1}{16})$ exponent

$=2+\frac{1}{32}$ multiply

$=\boxed{2\frac{1}{32}}=\boxed{\frac{65}{32}}=\boxed{2.0625}$

(21) The monthly revenue (in dollars) from selling x clocks is given by the polynomial $-0.3x^2+90x$.

a) Evaluate the polynomial for $x=200$.

b) Describe what $x=200$ and your result mean.

a) $-0.3(200)^2+90(200)$ ← replace x by (200)

$=-0.3(40000)+18000$ exp, mult

$=-12000+18000$

$=\boxed{\$6000}$

b) When the store sells 200 clocks, it collects $6000 from customers.